- **Perron-Frobenius Theorem**. If all $A_{ij} > 0$, then
  - There exists eigenvalue $\lambda^* > 0$ with algebraic multiplicity 1 such that for all other eigenvalues λ, $|\lambda| < \lambda^*$.
  - An eigenvector $\vec{v}^*$ associated with λ^* is strictly positive, i.e. $\vec{v}_i^* > 0$ for all i.
  - There are more parts to this theorem that you can find on Wikipedia.
- **Stochastic matrices** or **Markov matrices** are matrices that represent the transition probabilities in a Markov chain.
- In a stochastic matrix, A_{ij} represents the probability of transitioning to state j given that you are currently on state i. Hence, $0 \leq A_{ij} \leq 1$.
- Each row of a stochastic matrix must sum to 1.
- A^k just represents the transition matrix in k steps. This is just the Law of Total Probability.
- One curious observation to note is what happens to a stochastic matrix with a **steady state**, or the equilibrium A^∞, if one exists.
  - If an equilibrium exists, then note that $A^k = QD^kQ^T$, so the only eigenvalues that matter are when $\lambda = 1$ at $t = \infty$.
  - If $-1 < \lambda < 1$, then that eigenvalue will decay to 0, so we can ignore those.
  - If $\lambda \leq -1$ or $\lambda > 1$, then D^∞ doesn't converge, which means that equilibrium doesn't exist. In fact, the case that $\lambda > 1$ or $\lambda < -1$ is not possible, because that would imply that multiplying probabilities can explode.
  - The steady state is the eigenvector associated with $\lambda = 1$. This state does not change at each time step. Moreover, the probability vector will tend toward this steady state as $k \to \infty$.
  - The steady state and $\lambda = 1$ exists for all stochastic matrices where all $A_{ij} > 0$, or if there exists k such that all $A_{ij}^k > 0$.